**Chapter 4.5+ Paper**

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**4.5 - Normal Probability Distribution:**

The normal probability distribution produces a bell shaped graph that is connected to the empirical rule. Because the normal density function is symmetric around , the areas only need to be labeled on one side of the mean. Typically, the right side is chosen, and the points are marked as , which is the distance from the mean measured in standard deviations.

where:

and

Mean and Standard Deviation:

and

The normal random variable can be transformed into a standard normal random variable . locates a point that is measured from the mean of a normal random variable, expressed in units of standard deviation of the original normal random variable. Hence, the mean of is 0, and the standard deviation is 1.

**4.6 - Gamma Probability Distribution:**

Gamma probability distributions map bell shaped density functions that are skewed to one side. If is an integer, the function can be expressed as a sum of Poisson probabilities. However, if is not an integer, and , it is impossible to find a closed-form expression. Unless , it is impossible to obtain areas under gamma functions with direct integration.

where:

and

Mean and Standard Deviation:

and

is known as the gamma function. Direct integration shows that , and integration by parts shows that for any and that for any integer n. is the shape parameter, and dictates the shape of the curve. is the scale parameter, and dictates the size of the curse.

A chi-square distribution with degrees of freedom is a gamma-distributed random variable where and . Random variables with chi-square distributions are called chi-square random variables. The mean and standard deviation of a chi-square random variable are as follows:

and

If , then the gamma density function is called an exponential density function. Exponential density functions are used to model the life expectancy of electronic components.

Mean and Standard Deviation:

and

**4.7 - Beta Probability Distribution:**

Beta density functions are two-parameter density functions defined over a closed interval and is used to model proportions, such as the proportion of impurities in chemical products. The graphs of beta density functions assume varying shapes for different values of and . If , then defines a variable such that . This allows the beta density function to be applied to intervals that are not . It is defined as:

where:

Mean and Standard Deviation:

and

given that and

Incomplete Beta Function (cumulative distribution function):

where